Project 2 – Flight speeds at altitude

Updated March 27, 2014

2.1 Context and objectives

The objective of this project is to study the speed and altitude limits of a conventional aircraft in normal flight.

In the previous project you have calculated the three upper and lower limits on a jet aircraft's speed in steady flight. In the present project, we will quantify a fourth speed limit, and study the effect of altitude on these limits.

Based on your analysis of the aerodynamic data as well as atmospheric data,

- Establish the flight domain of the airplane, that is to say the entire range of speeds and altitudes at which it is able to remain in steady flight.
- Select a cruise altitude and speed, and calculate the corresponding required power, as well as the fuel consumption per kilometer traveled.

Your mark will be based on the clarity of your work as well as on the validity of your calculations. You may use any tool you wish (ex. software, books), but you are required to quote all of your sources.

Groups handing in written reports must hand in one single printed or PDF (A4-size) document, no longer than 8 pages.

Groups making an oral presentation must aim for less than 15 minutes (all members participating), and then answer questions from the class. Please hand in your slides as a print-out or PDF file.

2.2 Aircraft specifications

The aircraft is the same as for project 1 and data is presented in table 2.1. If you have already chosen a wing surface area, you may keep that value.

Crew	1 or 2
Capacity	5 to 6 pax
Length	12 m
Wingspan	12 m
Wing area	12 m^2
OWE	1 950 kg
MTOW	3 250 kg
Powerplant	2 × P&WC PW600, 4,5 kN each

Table 2.1: Specifications of the new aircraft

2.3 Engine data

The aircraft is powered by two Pratt & Whitney Canada PW610F turbofans. Each has a maximum thrust at sea level of:

$$T_{max,SL} = 4{,}5\,\mathrm{kN} \tag{2/1}$$

The engines provide thrust which decreases proportionally with the ambient air density and the throttle level.

Their specific fuel consumption is assumed constant,

$$SFC = 2{,}4 \cdot 10^{-4}\,\mathrm{s}^{-1} \tag{2/2}$$

Specific Fuel Consumption (SFC) is defined as the weight (in Newtons) of the fuel burned to generate one Newton of thrust during one second. It is measured in Newtons per Newton per second ($\mathrm{N\,N^{-1}\,s^{-1}}$), thus in $\mathrm{s^{-1}}$.

2.4 Aircraft aerodynamics

Wind tunnel measurements on a small model yield the values for aircraft lift coefficient (C_L) plotted in figure 2.1.

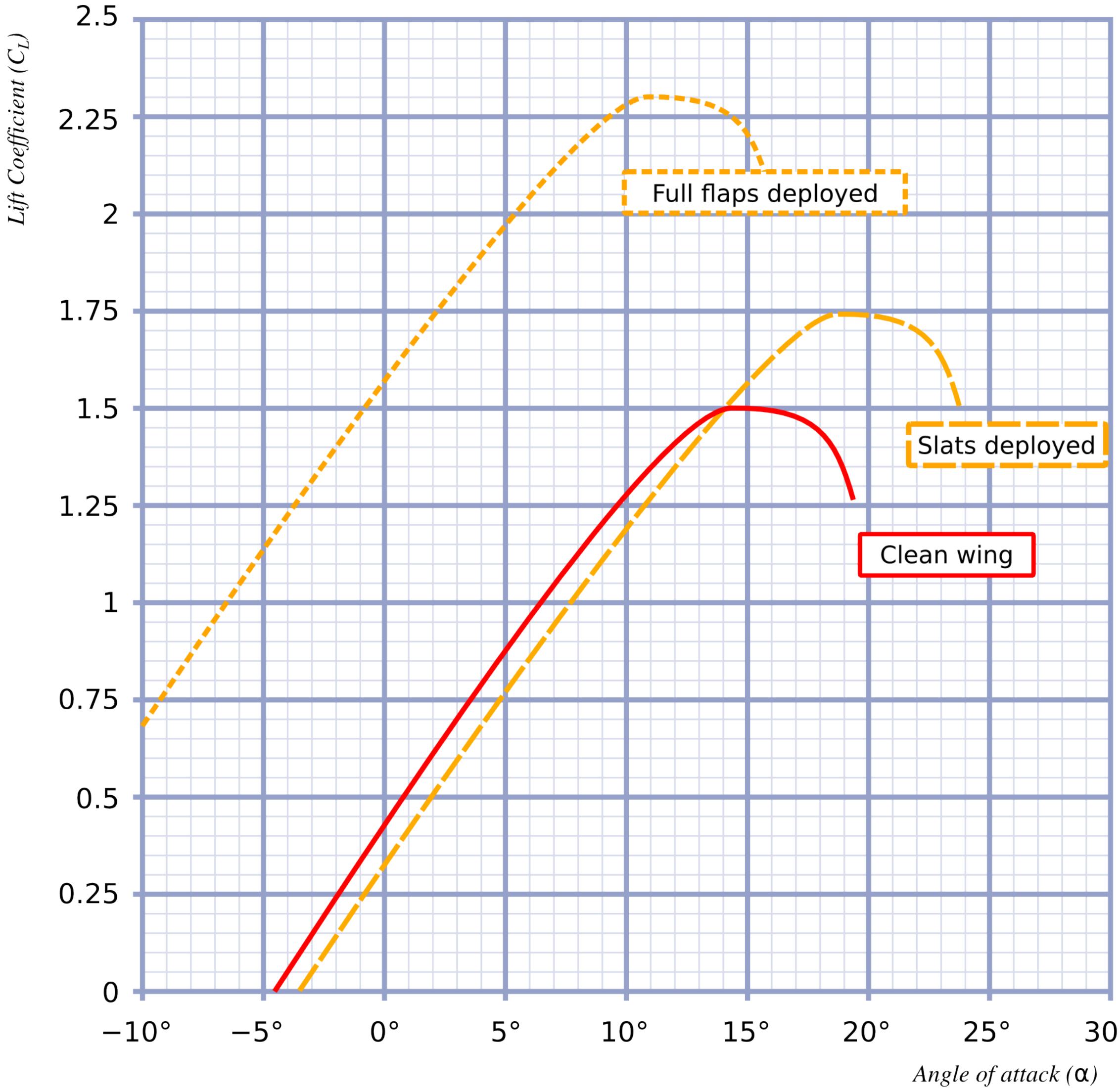

Figure 2.1: Lift coefficient as a function of angle of attack for the aircraft, as measured on a representative wind tunnel model.

Wind tunnel measurements also allowed aerodynamicists to model the drag coefficient as follows:

In clean wing configuration:

$$C_D = 0{,}024 + 0{,}044\ C_L^2 \qquad (2/3)$$

In landing configuration:

$$C_D = 0{,}056 + 0{,}05\ C_L^2 \qquad (2/4)$$

2.5 Mach number limit

The design maximum Mach number is 0,6. Beyond this value, shockwaves on the top surface of the wings abruptly decrease lift and increase drag.

2.6 Standard Atmosphere mathematical model

The atmospheric air properties can be predicted with the *International Standard Atmosphere* model:

Sea level temperature:

$$T_{SL} = 288{,}15\,\mathrm{K} \tag{2/5}$$

The temperature decreases with altitude according to the gradient T' (sometimes called *lapse rate*):

- $T' = -6{,}5\,\mathrm{K\,km^{-1}}$ from 0 to 11 km,
- $T' = 0\,\mathrm{K\,km^{-1}}$ from 11 to 20 km,
- $T' = +1\,\mathrm{K\,km^{-1}}$ from 20 to 32 km.

Sea level pressure:

$$p_{SL} = 1{,}01325 \cdot 10^5\,\mathrm{Pa} \tag{2/6}$$

With an elegant analysis of pressure distribution inside a static fluid, it can be shown that the atmospheric pressure p at any altitude z can be computed from the conditions at any point i according to the two relations 2/7 and 2/8 below:

When $T' \neq 0$:

$$p_{(z)} = p_{(z_i)} \left(1 + \frac{T'}{T_i}(z - z_i)\right)^{-\frac{g}{RT'}} \tag{2/7}$$

When $T' = 0$:

$$p_{(z)} = p_{(z_i)} \exp\left(-\frac{g(z - z_i)}{R\,T_i}\right) \tag{2/8}$$

Here atmospheric air is assumed to behave as a perfect gas with constant values $R = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and $\gamma = 1{,}4$.